PRINCIPLE OF MATHEMATICAL INDUCTION

1. For all positive integral values of $n, 3^{2n} - 2n + 1$ is divisible by
   a) 2 b) 4 c) 8 d) 12
2. $3 + 13 + 29 + 51 + 79 + \cdots$ to n terms =
   a) $2n^2 + 7n^3$ b) $n^2 + 5n^3$ c) $n^3 + 2n^2$ d) None of these
3. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction?
   a) $A^n = 2^{n-1}A + (n-1)I$ b) $A^n = nA + (n-1)I$
   c) $A^n = 2^{n-1}A - (n-1)I$ d) $A^n = nA - (n-1)I$
4. If $(n): 1 + 3 + 5 + \ldots + (2n-1) = n^2$ is
   a) True for all $n \in N$ b) True for $n > 1$ c) True for no n d) None of these
5. For a positive integer n, Let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{(2^n)-1}$. Then
   a) $a(100) \leq 100$ b) $a(100) > 100$ c) $a(200) \leq 100$ d) None of these
6. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then for $n \in N, A^n$ is equal to
   a) $\begin{bmatrix} \cos^n\theta & \sin^n\theta \\ -\sin^n\theta & \cos^n\theta \end{bmatrix}$ b) $\begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$ c) $\begin{bmatrix} n\cos\theta & n\sin\theta \\ -n\sin\theta & n\cos\theta \end{bmatrix}$ d) None of the above
7. The sum of the cubes of three consecutive natural numbers is divisible by
   a) 7 b) 9 c) 25 d) 26
8. For all $n \in N, \sum n$
   a) $< \dfrac{(2n+1)^2}{8}$ b) $> \dfrac{(2n+1)^2}{8}$ c) $= \dfrac{(2n+1)^2}{8}$ d) None of these
9. The product of three consecutive natural numbers is divisible by
   a) 5 b) 7 c) 6 d) 4
10. For all $n \in N, 5^{2n} - 1$ is divisible by
    a) 6 b) 11 c) 24 d) 26
11. $7^{2n} + 3^{n-1} . 2^{3n-3}$ is divisible by
    a) 24 b) 25 c) 9 d) 13
12. For $n \in N, 10^{n-2} \geq 81n$ is
    a) $n > 5$ b) $n \geq 5$ c) $n < 5$ d) $n > 8$
13. For all $n \in N, \frac{n^5}{5} + \frac{n^3}{3} + \frac{7}{15n}$ is
    a) An integer b) A natural number c) A positive fraction d) None of these
14. Let $S(k) = 1 + 3 + 5 \ldots + (2k-1) = 3 + k^2$. Then, which of the following is true?
    a) $S(1)$ is correct b) $S(k) \Rightarrow S(k+1)$
    c) $S(k) \not\Rightarrow S(k+1)$ d) Principle of mathematical induction can be used to prove the formula
15. The smallest positive integer n for which $n! < \left(\frac{n+1}{2}\right)^n$ holds, is
    a) 1 b) 2 c) 3 d) 4
16. The remainder when 5^{99} is divided by 13, is
    a) 6 b) 8 c) 9 d) 10

17. $10^n + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$

a) 7 b) 5 c) 9 d) 17

18. For all $n \in N, n^3 + 2n$ is divisible by

a) 3 b) 8 c) 9 d) 11

19. For all $n \in N, 7^{2n} - 48n - 1$ is divisible by

a) 25 b) 26 c) 1234 d) 2304

20. $10^n + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$

a) 7 b) 5 c) 9 d) 17

21. The nth term of the series $4 + 14 + 30 + 52 + 80 + 114 + \cdots$ is

a) $5n - 1$ b) $2n^2 + 2n$ c) $3n^2 + n$ d) $2n^2 + 2$

22. If $P(n)$ is a statement $(n \in N)$ such that, if $P(k)$ is true, $P(k+1)$ is true for $k \in N$, then $p(n)$ is true

a) For all n b) For all $n > 1$ c) For all $n > 2$ d) Nothing can be said

23. Using mathematical induction, then numbers $a_n's$ are defined by
a_0=1, $a_{n+1} = 3n^2 + n + a_n, (n \geq 0)$ Then, a_n is equal to

a) $n^3 + n^2 + 1$ b) $n^3 - n^2 + 1$ c) $n^3 - n^2$ d) $n^3 + n^2$

24. $\frac{(n+2)!}{(n-1)!}$ is divisible by

a) 6 b) 11 c) 24 d) 26

25. If $P(n) = 2 + 4 + 6 + \ldots + 2n, n \in N$, then $P(k) = k(k+1) + 2 \Rightarrow P(k+1) = (k+1)(k+2) + 2$ for all $k \in N$. So, we can conclude that $P(n) = n(n+1) + 2$ for

a) All $n \in N$ b) $n > 1$ c) $n > 2$ d) Nothing can be said

26. For all $n \in N$, $2 \cdot 4^{2n+1} + 3^{3n+1}$ is divisible by

a) 2 b) 9 c) 3 d) 11

27. For all $n \in N, n^4$ is less than

a) 10^n b) 4^n c) 5^n d) 10^{10}

28. The number $a^n - b^n$(a, b are distinct rational numbers and $n \in N$) is always divisible by

a) $a - b$ b) $a + b$ c) $2a - b$ d) $a - 2b$

29. If $n \in N$, then $3^{2n} + 7$ is divisible by

a) 3 b) 8 c) 9 d) 11

30. For each, $n \in N$, $10^{2n-1} + 1$ is divisible by

a) 11 b) 13 c) 9 d) None of these

31. If $10^n + 3.4^{n+2} + \lambda$ is exactly divisible by 9 for all $n \in N$, then the least positive integral value of λ is

a) 5 b) 3 c) 7 d) 1

32. For all $n \in N, \cos\theta \cos 2\theta \cos 4\theta \ldots \cos 2^{n-1}\theta$ equals to

a) $\frac{\sin 2^n \theta}{2^n \sin\theta}$ b) $\frac{\sin 2^n \theta}{\sin\theta}$ c) $\frac{\cos 2^n \theta}{2^n \cos 2\theta}$ d) $\frac{\cos 2^n \theta}{2^n \sin\theta}$

33. The inequality $n! > 2^{n-1}$ is true for

a) $n > 2$ b) $n \in N$ c) $n > 3$ d) None of these

34. If $P(n): 2 + 4 + 6 \ldots + (2n), n \in N$, then
$P(k) = k(k+1) + 2$implies
$P(k) = (k+1)(k+2) + 2$
is true for all $k \in N$. So, statement $P(n) = n(n+1) + 2$ is true for

a) $n \geq 1$ b) $n \geq 2$ c) $n \geq 3$ d) None of these

35. If $P(n): 3^n < n!, n \in N$, then $P(n)$ is true

a) For $n \geq 6$ b) For $n \geq 7$ c) For $n \geq 3$ d) For all n

36. Let $P(n)$ denotes the statement that $n^2 + n$ is odd. It is seen that $P(n) \Rightarrow P(n+1), P(n)$ is true for all

a) $n > 1$ b) n c) $n > 2$ d) None of these

37. The sum to n terms of the series $1^3 + 3^3 + 5^3 + \cdots$ is

a) $n^2(n^2 - 1)$ b) $n^2(2n^2 - 1)$ c) $n^2(2n^2 + 1)$ d) $n^2(n^2 + 1)$

38. If $n \in N$, then $11^{n+2} + 12^{2n+1}$ is divisible by

a) 113 b) 123 c) 133 d) None of these

39. For natural number $n, 2^n(n-1)! < n^n$, if

a) $n < 2$ b) $n > 2$ c) $n \geq 2$ d) never

40. If $n \in N$, then $n(n^2 - 1)$ is divisible by

a) 6 b) 16 c) 36 d) 24

41. $(2^{3n} - 1)$ will be divisible by $(\forall n \in N)$

a) 25 b) 8 c) 7 d) 3

42. If $n \in N$, then $x^{2n-1}y^{2n-1}$ is divisible by

a) $x + y$ b) $x - y$ c) $x^2 + y^2$ d) None of these

43. If $x^{2n-1} + y^{2n-1}$ is divisible by $x + y$, if n is

a) A positive integer
b) An even positive integer
c) An odd positive integer
d) None of the above

44. If m, n are any two odd positive integer with $n < m$, then the largest positive integers which divides all the numbers of the type $m^2 - n^2$ is

a) 4 b) 6 c) 8 d) 9

45. If $x^n - 1$ is divisible by $x - k$, then the least positive integral value of k is

a) 1 b) 2 c) 3 d) 4

46. If n is a positive integer, then $5^{2n+2} - 24n - 25$ isdivisible by

a) 574 b) 575 c) 675 d) 576

47. For all $n \in N, 3^{3n} - 26^n - 1$ is divisible by

a) 24 b) 64 c) 17 d) 676

48. Matrix A is such that $A^2 = 2A - I$ where I is the identity matrix, then for $n \geq 2, A^n$ is equal to

a) $n\,A - (n-1)I$ b) $n\,A - I$ c) $2^{n-1}A - (n-1)I$ d) $2^{n-1}A - I$

49. If $a_1 = 1$ and $a_n = na_{n-1}$ for all positive integer $n \geq 2$, then a_5 is equal to

a) 125 b) 120 c) 100 d) 24

50. For all $n \in N, 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}}$ is

a) Equal to $\sqrt{n}$
b) Less than or equal to $\sqrt{n}$
c) Greater than or equal to $\sqrt{n}$
d) None of these

51. Let $P(n): n^2 + n + 1$ is an even integer. If $P(k)$ is assumed true $\Rightarrow P(k+1)$ is true. Therefore, $P(n)$ is true

a) For $n > 1$ b) For all $n \in N$ c) For $n > 2$ d) None of these

52. $2^{3n} - 7n - 1$ is divisible by

a) 64 b) 36 c) 49 d) 25

53. For all $n \in N, 3n^5 + 5n^3 + 7n$ is divisible by

a) 3 b) 5 c) 10 d) 15

54. If n is a positive integer, then $n^3 + 2n$ is divisible by

a) 2 b) 6 c) 15 d) 3

55. For all $n \in N, 49^n + 16n - 1$ is divisible by

a) 64 b) 8 c) 16 d) 4

56. If $P(n)$ is a statement such that $P(3)$ is true. Assuming $P(k)$ is true $\Rightarrow P(k+1)$ is true for all $k \geq 3$, then $P(n)$ is true

a) For all n b) For $n \geq 3$ c) For $n > 4$ d) None of these

57. If n is an odd positive integer, then $a^n + b^n$ is divisible by

a) $a + b$ b) $a - b$ c) $a^2 + b^2$ d) None of these

58. The nth terms of the series $3 + 7 + 13 + 21 + \cdots$ is

a) $4n - 1$ b) $n^2 + 2n$ c) $n^2 + n + 1$ d) $n^2 + 2$

59. If a, b are distinct rational numbers, then for all $n \in N$ the number $a^n - b^n$ is divisible by

a) $a - b$ b) $a + b$ c) $2a - b$ d) $a - 2b$

60. $x(x^{n-1} - n\alpha^{n-1}) + \alpha^n(n-1)$ is divisible by $(x-\alpha)^2$ for

a) $n > 1$ b) $n > 2$ c) All $n \in N$ d) None of these

61. $2^{3n} - 7n - 1$ is divisible by

a) 64 b) 36 c) 49 d) 25

62. The sum of n terms of the series $1 + (1+a) + (1+a+a^2) + (1+a+a^2+a^3) + \cdots$, is

a) $\frac{n}{1-a} - \frac{a(1-a^n)}{(1-a)^2}$ b) $\frac{n}{1-a} + \frac{a(1-a^n)}{(1-a)^2}$ c) $\frac{n}{1-a} + \frac{a(1+a^n)}{(1-a)^2}$ d) $-\frac{n}{1-a} + \frac{a(1-a^n)}{(1-a)^2}$

63. For each $n \in N$, $3^{2n} - 1$ is divisible by

a) 8 b) 16 c) 32 d) None of these

64. If n is an even positive integer, then $a^n + b^n$ is divisible by

a) $a + b$ b) $a - b$ c) $a^2 - b^2$ d) None of these

65. The greatest positive integer, which divides $(n+2)(n+3)(n+4)(n+5)(n+6)$ for all $n \in N$, is

a) 4 b) 120 c) 240 d) 24

66. If $3 + 5 + 9 + 17 + 33 + \cdots$ to n terms $= 2^{n+1} + n - 2$, then nth term of LHS is

a) $3^n - 1$ b) $2n + 1$ c) $2^n + 1$ d) $3n - 1$

67. For all $n \in N$, $10^n + 3.4^{n+2} + 5$ is divisible by

a) 23 b) 3 c) 9 d) 207

68. For all $n \in N$, $4^n - 3n - 1$ is divisible by

a) 3 b) 8 c) 9 d) 11

PRINCIPLE OF MATHEMATICAL INDUCTION

: ANSWER KEY :

1)	a	2)	c	3)	d	4)	a	41)	c	42)	a	43)	a	44)	c
5)	a	6)	b	7)	b	8)	a	45)	a	46)	d	47)	d	48)	a
9)	c	10)	c	11)	b	12)	b	49)	b	50)	b	51)	d	52)	c
13)	b	14)	b	15)	b	16)	b	53)	d	54)	d	55)	a	56)	b
17)	c	18)	a	19)	d	20)	c	57)	a	58)	c	59)	a	60)	c
21)	c	22)	d	23)	b	24)	a	61)	c	62)	a	63)	a	64)	d
25)	d	26)	d	27)	a	28)	a	65)	b	66)	c	67)	c	68)	c
29)	b	30)	a	31)	a	32)	a								
33)	a	34)	d	35)	b	36)	d								
37)	b	38)	c	39)	b	40)	a								

PRINCIPLE OF MATHEMATICAL INDUCTION

: HINTS AND SOLUTIONS :

1 **(a)**
On putting $n=2$ in $3^{2n}-2n+1$, we get
$3^{2\times 2}-2\times 2+1=81-4+1=78$
Which is divisible by 2

2 **(c)**
Clearly, n^3+2n^2 gives the sum of the series for $n=1,2,3$ etc.

3 **(d)**
$A^2=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$
$A^3=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$
$A^n=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ can be verified by induction. Now, taking option
(b) $\begin{bmatrix}1 & 0\\ n & 1\end{bmatrix}=\begin{bmatrix}n & 0\\ n & n\end{bmatrix}+\begin{bmatrix}n-1 & 0\\ 0 & n-1\end{bmatrix}$
$\Longrightarrow \begin{bmatrix}1 & 0\\ n & 1\end{bmatrix}\neq\begin{bmatrix}2n-1 & 0\\ 1 & 2n-1\end{bmatrix}$
(d)$nA-(n-1)I=\begin{bmatrix}n & 0\\ n & n\end{bmatrix}-\begin{bmatrix}n-1 & 0\\ 1 & n-1\end{bmatrix}$
$=\begin{bmatrix}1 & 0\\ n & 1\end{bmatrix}=A^n$

4 **(a)**
Given, $(n): 1+3+5+\ldots+(2n-1)=n^2$
$P(1): 1=1$(true)
Let $P(k)=1+3+5+\ldots+(2k-1)=k^2$
$\therefore P(k+1)=1+3+5+\ldots+(2k-1)+2k+1$
$=k^2+2k+1=(k+1)^2$
So, it holds for all n.

5 **(a)**
It can be proved with the help of mathematical induction that $\frac{n}{2}<a(n)\leq n$.
$\therefore \dfrac{200}{2}<a(200)\Rightarrow a(200)>100$
and $a(100)\leq 100$

6 **(b)**
Given, $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$
Now, $A^2=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$
$=\begin{bmatrix}\cos^2\theta-\sin^2\theta & 2\sin\theta\cos\theta\\ -2\sin\theta\cos\theta & \cos^2\theta-\sin^2\theta\end{bmatrix}$
$=\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$
$\therefore$ By induction, $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$

9 **(c)**
Let $P(n)\equiv n(n+1)(n+2)$
$P(1)\equiv 1\cdot 2\cdot 3=6$
$P(2)\equiv 2\cdot 3\cdot 4=24$
Hence, it is divisible by 6.

10 **(c)**
We have,
$5^{2n}-1=(5^2)^n-1=(1+24)^n-1$
$\Rightarrow 5^{2n}-1={}^nC_1\times 24+{}^nC_2\times 24^2+\cdots+{}^nC_n\times 24^n$
$\Rightarrow 5^{2n}-1=24({}^nC_1+{}^nC_2\times 24+\cdots+{}^nC_n\times 24^{n-1})$
$\Rightarrow 5^{2n}-1$ is divisible by 24 for all $n\in N$

12 **(b)**
Let $P(n): 10^{n-2}\geq 81n$
For $n=4, 10^2\ngeq 81\times 4$
For $n=5, 10^3\geq 81\times 5$
Hence, by mathematical induction for $n\geq 5$, the proposition is true.

14 **(b)**
$S(k)=1+3+5....+(2k-1)=3+k^2$
Put $k=1$ in both sides, we get
LHS $=1$ and RHS$=3+1=4$
$\Longrightarrow$ LHS $\neq$ RHS
Put $(k+1)$ in both sides in the place of k, we get
LHS $=1+3+5....+(2k-1)+(2k+1)$
RHS $=3+(k+1)^2=3+k^2+2k+1$
Let LHS $=$ RHS
Then,$1+3+5....+(2k-1)+(2k+1)$
$=3+k^2+2k+1$
$\Longrightarrow 1+3+5+...+(2k-1)=3+k^2$
If $S(k)$ is true, then $S(k+1)$ is also true.
Hence, $S(k)\Longrightarrow S(k+1)$

15 **(b)**
Given, $n!<\left(\frac{n+1}{2}\right)^n$

At $n = 1$,
$1! \nless 1$
At $n = 2$,
$2! < \left(\frac{3}{2}\right)^2$
$\Rightarrow 2 < 2.25$ which is true.

16 **(b)**
$5^{99} = 5(5^2)^{49} = 5(25)^{49}$
$= 5(26 - 1)^{49}$
$= 5 \times 26 \times (\text{Positive term}) - 5$
So, when it is divided by 13, it gives the remainder -5 or 8.

17 **(c)**
On putting $n = 2$ in $10^n + 3(4^{n+2}) + 5$, we get
$10^2 + 3 \times 4^4 + 5 = 100 + 768 + 5 = 873$
Which is divisible by 9

18 **(a)**
For $n = 1, 2, 3$, we find that $n^3 + 2n$ takes values 3, 12 and 33, which are divisible by 3

19 **(d)**
We have,
$7^{2n} - 48n - 1 = (1 + 48)^n - 48n - 1$
$\Rightarrow 7^{2n} - 48n - 1$
$= {}^nC_2 \times 48^2 + {}^nC_3 \times 48^3 + \cdots + {}^nC_n \times 48^n$
$\Rightarrow 7^{2n} - 48n - 1$ divisible by 48^2 i.e., 2304

20 **(c)**
For $n = 1, 10^n + 3 \cdot 4^{n+2} + 5$
$= 10 + 3 \cdot 4^3 + 5 = 207$ which is divisible by 9.
$\therefore$ By induction, the result is divisible by 9.

21 **(c)**
We observe that $3n^2 + n$ gives various terms of the series by putting $n = 1, 2, 3, \ldots$

22 **(d)**
Unless we prove $P(1)$ is true, nothing can be said.

23 **(b)**
Given, $a_0 = 1, a_{n+1} = 3n^2 + n + a_n$
$\Longrightarrow a_1 = 3(0) + 0 + a_0 = 1$
$\Longrightarrow a_2 = 3(1)^2 + 1 + a_1 = 3 + 1 + 1 = 5$
From option (b),
Let $P(n) = n^3 - n^2 + 1$
$\therefore \quad P(0) = 0 - 0 + 1 = 1 = a_0$
$P(1) = 1^3 - 1^2 + 1 = 1 = a_1$
and $P(2) = (2)^3 - (2)^2 + 1 = 5 = a_2$

25 **(d)**
It is obvious, nothing can be said.

26 **(d)**
Let $P(n) = 2 \cdot 4^{2n+1} + 3^{3n+1}$
$P(1) \equiv 128 + 81 = 209$(divisible by 11 only)

28 **(a)**
Let $P(n) \equiv a^n - b^n$
$P(1) \equiv a - b$
$P(2) \equiv a^2 - b^2$
Hence, it is divisible by $a - b$.

29 **(b)**
$3^{2n} + 7$ is divisible by 8. This can be checked by putting $n = 1, 2, 3$ etc.

30 **(a)**
Let $P(n) = 10^{2n-1} + 1$
$P(1) = 10 + 1 = 11$
Let $P(k) \equiv 10^{2k-1} + 1 = 11I$ is true
Now, $P(k + 1) = 10^{2k+1} + 1$
$= (11I - 1)100 + 1$
$= 1100I - 99 = 11I_1$
So, $P(k + 1)$ is true.

33 **(a)**
Let $P(n) \equiv n! > 2^{n-1}$
$P(3) \equiv 6 > 4$
Let $P(k) \equiv k! > 2^{k-1}$ is true.
$\therefore P(k + 1) = (k + 1)! = (k' + 1)k!$
$> (k + 1)2^{k-1}$
$> 2^k \quad (\text{as } k + 1 > 2)$

34 **(d)**
Here, $P(1) = 2$ and from the equation
$P(k) = k(k + 1) + 2$
$\Longrightarrow \quad P(1) = 4$
So, $P(1)$ is not true
Hence, mathematical induction is not applicable.

35 **(b)**
Given that, $P(n)$: $3^n < n!$
Now, $P(7)$: $3^7 < 7!$ is true
Let $P(k)$: $3^k < k!$
$\Rightarrow P(k + 1): 3^{k+1} = 3 \cdot 3^k < 3 \cdot k! < (k + 1)! \quad (\because k + 1 > 3)$

36 **(d)**
$P(n) = n^2 + n$
It is always odd but square of any odd number is always odd and also sum of two odd number is always even. So, for no any n for which this statement is true.

37 **(b)**
$n^2(2n^2 - 1)$ gives the sum of the series for $n = 1, 2,$ etc.

38 **(c)**
On putting $n = 1$ in $11^{n+2} + 12^{2n+1}$, we get
$11^{1+2} + 12^{2\times1+1} = 11^3 + 12^3 = 3059$
Which is divisible by 133

39 **(b)**

The condition $2^n(n-1)! < n^n$ is satisfied for $n > 2$

40 **(a)**

We have,

$n(n^2-1) = (n-1)(n+1)$, which is product of three consecutive natural numbers and hence divisible by 6

41 **(c)**

$2^{3n} - 1 = (2^3)^n - 1$

$= 8^n - 1 = (1+7)^n - 1$

$= 1 + {}^nC_1 7 + {}^nC_2 7^2 + ... + {}^nC_n 7^n - 1$

$= 7[{}^nC_1 + {}^nC_2 7 + ... + {}^nC_n 7^{n-1}]$

$\therefore 2^{3n} - 1$ is divisible by 7

43 **(a)**

Let $P(n) \equiv x^{2n-1} + y^{2n-1} = \lambda(x+y)$

$P(1) \equiv x + y = \lambda_1(x+y)$

$P(2) \equiv x^3 + y^3 = \lambda_2(x+y)$

Hence, for $\forall\, n \in N, P(n)$ is true.

44 **(c)**

Let $m = 2k+1, n = 2k-1\ (k \in N)$

$\therefore m^2 - n^2 = 4k^2 + 1 + 4k - 4k^2 + 4k - 1 = 8k$

Hence, All the numbers of the form $m^2 - n^2$ are always divisible by 8.

46 **(d)**

Let $P(n) = 5^{2n+2} - 24n - 25$

For $\quad n = 1$

$P(1) = 5^4 - 24 - 25 = 576$

$P(2) = 5^6 - 24(2) - 25 = 15552$

$= 576 \times 27$

Here, we see that $P(n)$ is divisible by 576

47 **(d)**

We have,

$3^{3n} - 26n - 1 = 27^n - 26n - 1$

$\Rightarrow 3^{3n} - 26n - 1 = (1+26)^n - 26n - 1$

$\Rightarrow 3^{3n} - 26n - 1$

$= {}^nC_2 \times 26^2 + {}^nC_3 \times 26^3 + \cdots + {}^nC_n \times 26^n$

Clearly, RHS is divisible 26^2 i.e. 676

48 **(a)**

As we have $A^2 = 2A - I$

$\Rightarrow \quad A^2 A = (2A - I)A = 2A^2 - IA$

$\Rightarrow \quad A^3 = 2(2A - I) - IA = 3A - 2I$

Similarly, $A^4 = 4A - 3I$

$A^5 = 5A - AI$

$A^n = nA - (n-1)I$

49 **(b)**

Given, $a_n = n a_{n-1}$

For $\quad n = 2$

$a_2 = 2a_1 = 2 \quad (\because a_1 = 1 \text{given})$

$a_3 = 3a_2 = 3(2) = 6$

$a_4 = 4(a_3) = 4(6) = 24$

$a_5 = 5(a_4) = 5(24) = 120$

51 **(d)**

Given, $P(n): n^2 + n + 1$

At $n = 1, P(1) : 3$, which is not an even integer.

$\therefore P(1)$ is not true (Principle of Induction is not applicable).

Also, $n(n+1) + 1$ is always an odd integer.

52 **(c)**

Let $P(n) = 2^{3n} - 7n - 1$

$\therefore \quad P(1) = 0, P(2) = 49$

$P(1)$ and $P(2)$ are divisible by 49.

Let $P(k) \equiv 2^{3k} - 7k - 1 = 49I$

$\therefore P(k+1) \equiv 2^{3k+3} - 7k - 8$

$= 8(49I + 7k + 1) - 7k - 8$

$= 49(8I) + 49k = 49I_1$

Alternate

$P(n) = (1+7)^n - 7n - 1$

$= 1 + 7n + 7^2 \dfrac{n(n-1)}{2!} + \cdots - 7n - 1$

$= 7^2 \left(\dfrac{n(n-1)}{2!} + \cdots\right)$

53 **(d)**

Putting $n = 1, 2, 3 ...$, it can be checked that $3n^5 + 5n^3 + 7n$ is divisible by 15

54 **(d)**

Let $P(n) = n^3 + 2n$

$\Longrightarrow P(1) = 1 + 2 = 3$

$\Longrightarrow P(2) = 8 + 4 = 12$

$\Longrightarrow P(3) = 27 + 6 = 33$

Here, we see that all these number are divisible by 3

55 **(a)**

We observe that $49^n + 16n - 1$ takes values 64

Hence, $49^n + 16n - 1$ is divisible by 64

56 **(b)**

Since, $P(3)$ is true.

Assume $P(k)$ is true $\Rightarrow P(k+1)$ is true means, if $P(3)$ is true $\Rightarrow P(4)$ is true $\Rightarrow P(5)$ is true and so on. So, statement is true for all $n \geq 3$.

58 **(c)**

Putting $n = 1, 2, 3 ...$, we observe that $4n - 1$ is the nth term

60 **(c)**

Let $P(n) = x(x^{n-1} - n\alpha^{n-1}) + \alpha^n(n-1) = (x-\alpha)^2 g(x)$

$P(1) \equiv 0$ is true.

Let $P(k)$ is true.

$ie, x\left(x^{k-1} - k\alpha^{k-1}\right) + \alpha^k(k-1) = (x-\alpha)^2 g(x)$

Now, $P(k+1) \equiv x[x^k - (k+1)\alpha^k] + \alpha^{k+1}(k)$

$\equiv (x-\alpha)^2[xg(x)+k\alpha^{k-1}]$ (True)

So, holds for all $n \in N$.

61 **(c)**

Let $P(n) = 2^{3n} - 7n - 1$

$\therefore P(1) = 0$

$P(2) = 49$

$P(1)$ and $P(2)$ are divisible by 49.

Let $P(k) \equiv 2^{3k} - 7k - 1 = 49I$

$\therefore P(k+1) \equiv 2^{3k+3} - 7k - 8$

$= 8(49I + 7k + 1) - 7k - 8$

$= 49(8I) + 49k = 49I_1$

Hence, by mathematical induction $2^{3n} - 7n - 1$ is divisible by 49.

63 **(a)**

Let $P(n) = 3^{2n} - 1$

At $n = 1, P(1) = 8$ which is divisible by 8.

$\therefore P(1)$ is true.

Let $P(k)$ is true, then

$P(k) \equiv 3^{2k} - 1 = 8I$

$\therefore P(k+1) \equiv 3^{2k+2} - 1 = (8I+1)9 - I$

$= 72I + 8 = 8I_1$

$\therefore P(n)$ is divisible by 8, $\forall\, n \in N$.

68 **(c)**

It can be checked that $4^n - 3n - 1$ is divisible by 9 for $n = 1, 2, 3, \dots$